THE EFFECT OF DIURNAL VARIATION OF CLOCK RATES UPON LONGITUDE WORK.

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(*Read April 21, 1922.*)

In dealing with the question of a diurnal variation of clock rates it may be necessary to introduce some reservations—to employ the language of recent diplomatic conferences.

These reservations are mainly covered by treating the phenomena as those of observation, and still further limiting the data of observation to the meridian circle transits of stars.

Either our clocks run faster at night or there are systematic corrections to our observations that have not been detected nor applied.

Such systematic corrections might be due to errors of observation and reduction, or to some periodic term affecting the position of the meridian.

The hourly rate of a clock, computed from transit observations during any period of a night, will always differ from the average hourly rate during a period of one day unless, by a rare chance, the accidental error of observation exactly balances the error of the adopted right ascensions.

These two classes of error are of nearly the same order of magnitude as regards their accidental character.

For instance, the right ascensions of NEWCOMB, as tabulated in the *American Ephemeris,* have average accidental errors of at least $\pm 0^s.02$ per star.

The probable error of an observation with our instrument is quite precisely $\pm 0^s.02$.

With any number of stars used, the probable error of an observed clock correction, and that of an observed rate, would be made up of virtually equal errors of observation and of right ascension. Of yet greater importance in deriving hourly rates are the systematic errors of the adopted right ascensions, since these can not be diminished by increasing the number of stars observed.

Both classes of error can be eliminated by observing the same stars in each hour of right ascension, and closing a cycle of observations in which every hour has been included. Also some systematic errors of observation, such as that due to magnitude equation, are eliminated in the cycle.

The average systematic error of NEWCOMB's right ascensions at this date is not far from $\pm 0^s.02$ per hour.

The average difference between two consecutive hours is smaller than the average per hour, since the systematic errors are periodic in character, approximately of the form, $-0^s.02 \cos \alpha + 0^s.01 \sin \alpha$. A comparison of the right ascensions of NEWCOMB in the *American Ephemeris* with those of Boss in his *Preliminary General Catalogue* gives an indication of the character of the systematic errors to be anticipated.

For 325 stars at present under observation here, between 37° north declination and 30° south, the average difference is $\pm 0^s.022$ per star. In hourly groups the average difference is $\pm 0^s.006$ per hour. Differences between individual stars are evidently mainly fortuitous.* This does not imply an absence of systematic errors, but does indicate that the systematic errors of the two authorities are similar. The average difference between two consecutive hours is $\pm 0^s.010$ for the two lists.

If we use the right ascensions of Boss, the computed hourly rate of a clock will differ $0^s.010$ from the hourly rate computed with NEWCOMB's right ascensions, in the average, and may differ more than twice that amount. The right ascensions of Boss appear to have the relative weight 3 : 1, as regards accidental errors, and weight 2 : 1, for systematic errors.

The later fundamental system of AUWERS appears to be as precise as that of Boss, both as regards accidental errors and systematic errors.

NEWCOMB's system antedates the other two by about ten years. If we include as clock stars only those within 15° of the equator, 317 in number, the average differences between NEWCOMB and Boss are

* The difference $0^s.006$ for an average of 13 stars per hour corresponds to the average difference of $0^s.022$ per star.

nearly the same as those above, but the mean difference is $B - N = -0^s.009$, while the mean difference of the more widely extended list is less than $0^s.001$. Many of these stars close to the equator were not included in our observing program, since the intervals between successive stars were often too short.

Boss P. G. C.—Newcomb A. E.

R. A.	Stars	$\Delta\ \alpha$	Diff.
0	11	$-0^s.003$	
			$0^s.018$
1	13	+ 015	
			015
2	15	000	
			016
3	15	− 016	
			012
4	13	− 004	
			005
5	16	+ 001	
			005
6	12	− 004	
			003
7	14	− 001	
			011
8	14	− 012	
			025
9	11	+ 013	
			001
10	11	+ 014	
			015
11	15	− 001	
			008
12	11	+ 007	
			002
13	11	+ 005	
			013
14	13	− 008	
			008
15	15	000	
			005
16*	15	− 005	
			004
17	9	− 001	
			003
18	15	+ 002	
			005

19	13	— 003	
			016
20	15	— 019	
			022
21	14	+ 003	
			011
22	16	— 008	
			013
23	18	+ 005	
			008
Average	13	± 0.006	± 0.010

From our observations extending over a period of a quarter of a century the mean excess of the hourly rate at night over the average hourly rate during one day is $0^s.006$. This corresponds to a variation in the daily rate of an amplitude of approximately 0.3 of a second. The difference between observed and interpolated clock corrections would be a maximum for an interval of six hours, and would amount to over $0^s.03$. Double this difference would occur between the observations near sunset and sunrise, and an observed difference of $0^s.06$ has been found between clock corrections at those epochs of the day. These numerical results are still subject to revision, as more precise values are to be anticipated from our current series of observations.

In fundamental right-ascension observations differences of this size should occur, but the alternate observations of groups of stars, twelve hours apart, has smoothed out this effect in our adopted systems.

It is not often necessary to carry the daily rate forward more than two hours except in fundamental work, and the difference between daily and hourly rates would rarely introduce an error exceeding $0^s.01$.

It has been our custom generally to adopt the hourly rate derived during the period of observation, when that period is of sufficient length, in reducing transit observations. The results thus obtained conform to the adopted right-ascension system, with its errors included. The actual performance of the clock has been of secondary importance in deriving the right ascensions.

* [one star] ω *Herculis,* mistake in P.M.

Changes of temperature in our well-protected clock cases have had no sensible effect upon daily clock rates for short periods, such as one day, with which we are here concerned.

Ordinarily the range is less than one degree, and nearly always it is progressive, so that the temperature at night falls between those of successive days.

The variation of atmospheric pressure, as recorded by the barometer, affects the rate of a pendulum clock not hermetically sealed. For our Riefler clock, installed in 1907, a change of one inch in atmospheric pressure changes the daily rate $0^s.46$.

In our fine summer weather the average barometer reading at midnight is 0.04 inch below that at noon, and the lowest reading commonly occurs in the early morning, following midnight. In fine weather in winter the reading at midnight is 0.01 inch below that at noon, and the lowest readings occur in the afternoon.

The summation of the hourly excess at night has been divided into two periods, corresponding to the use of the Dent clock, unsealed up to 1907, and the use of the Riefler, following that date.

The first period gives a mean hourly excess of $0^s.007$, and the second period gives $0^s.004$. The difference between the two results and the mean results are too large to be accounted for by the variation of the atmospheric pressure.

The observation of clock corrections and rates during the night hours should be uninfluenced by any possible deviation of the meridian plane, due to barometric or thermal gradients in the atmosphere, such as might be suspected at sunset and sunrise.

The effect of such gradients has been found to be very small, even at these epochs of greatest disturbance.

EXCESS OF HOURLY RATES AT NIGHT.

Series	$\Delta\rho$	Wt.
1893 to 1894	$-0^s.0097$	2
94 " 95	-0.0059	2
95 " 96	-0.0066	1
97 " 98	-0.0039	2
98 " 1900	-0.0076	4
1901 " 04	-0.0086	4
05 " 06	-0.0052	2
06 " 07	-0.0047	2

07 " 08	—0.0037	1
14 " 15	—0.0038	3
17 " 18	—0.0047	3
1920	—0.0037	3
Mean (12)	—0.0057	
	±0.0004	
Weights	—0.0059	

The weights have been assigned according to the number of nights per year, and the number of stars per night.

Small undetected progressive changes in the position of the instrument would be represented in the computed clock corrections and rates. The changes in instrumental corrections were usually measured over a period of at least four hours.

It is hardly credible that uniform progressive changes would persist undetected in this long term of years if they were of sufficient size to account for the hourly rates as observed.

To explain the sunset and sunrise results, systematic differences would need to be of a decided character. Instrumental corrections and the indications given by the mire readings have had careful scrutiny in this connection.

Physical or mental fatigue might be presumed to affect the personal equation of the observer, and thus influence the computed clock rate. The effect would more probably produce erratic results, with larger accidental errors of observation.

The reaction times at sunset and sunrise would necessarily be of quite different character, also, from those at night to make plausible this explanation. Our current series of observations will give a test of such a possible effect, as we shall have mean hourly excesses during periods of six consecutive hours, in each of which the systematic errors of right ascension will have been eliminated.

A diurnal term in longitude, similar in character to the fourteen-month variation, would produce a diurnal periodic variation in clock corrections and rates, as observed. If the maxima occur at sunset and sunrise, the most rapid changes would occur at noon and midnight.

Clocks do not run over long periods of time with the uniformity requisite to test the fourteen-month term, and we derive the long-term variations in the longitude from the corresponding observed variations of latitude. But any good clock can be relied upon during a period of one day to test a diurnal variation.

If there is a physical cause for a diurnal variation in our observed results, the best clocks will give the best defined variations.

Since latitude observations with the zenith telescope have been confined to the night hours, we can not expect much contribution to the solution of a diurnal term from that delicate differential instrument. If the maxima of longitude variation occur at sunset and sunrise, the maxima of latitude variation should come at noon and midnight.

The current observations at the international zenith telescope stations are made in two groups, at nearly equal intervals each side of midnight.

The closing error of the groups, which is about 0″.2 distributed among twelve periods, might be due to a variation with a daily maximum that does not fall exactly half way between the two daily groups.

At your neighboring institution, the Flower Observatory of the University of Pennsylvania, a distinct difference in latitude results was derived by Prof. C. L. Doolittle between early and late hours of the night. This difference could be attributed to an error in the adopted constant of aberration, and a correction (0″.08) was computed by that most thorough and capable observer. It will be recalled that the zenith telescope observations have pretty uniformly given larger values for the constant of aberration than those derived from other sources. With the value of the constant, 20″.47, only one quarter of the difference derived by Prof. Doolittle would be accounted for.

Observations with the prime vertical instrument by M. Jean Boccardi, at the Turin Observatory, in 1920, were designed to show a differential effect in latitude results during the night hours.

By comparing the observed differences between stars separated about three hours in interval, through a cycle of nearly one year, he derived a cosine term with an average coefficient of 0″.07.

The extensive series of prime vertical observations of *a Lyræ* at the U. S. Naval Observatory, seventeen hundred observations in nineteen years, exhibit a difference of $0''.5$ between declinations measured by day and night.

Corrections to the adopted constant of nutation were computed from this series, from both day and night observations combined, and from the two periods of the day separately.

The first of the solutions mentioned gives a correction of $+0''.03$ to the constant $9''.22$.

To return to meridian circle results, our fundamental work during the years 1905 to 1908, and in 1916, has given us the observed latitude at all hours of day and night.

Over one thousand observations have been combined in deriving the following diurnal term. More than one quarter of the total number are of zenith stars, close enough to the zenith point to be observed facing either north or south for the measurement of bisection error.

A somewhat larger number are of stars bright enough for daylight observation, divided into groups for which the means are close to the zenith.

The stars α *Andromedæ* and *Polaris* furnish a third of the total number.

Observations of *Polaris* and β *Ursæ Minoris* are the only ones for which corrections for the diurnal variation of refraction are of importance. This correction has been derived from observations of stars at large zenith distances on both sides of the zenith, and the solution is independent of the latitude and its variation and of the nadir readings. The diurnal variation in the atmospheric refraction at this station is approximately one per cent. of the total refraction. A separate solution for the diurnal term had been made from the zenith stars only, before including the results from the other stars. Errors of refraction could play no part in this solution, which gave the same numerical coefficient as the solution from all the stars, within one unit in the third place of decimals.

The observations are nearly evenly distributed between daylight and night hours.

All have been corrected for the latitude variation of long periods from the results of the international zenith telescope stations, excluding the *z* term.

The diurnal term given by these observations is $+0''.14 \cos T$, where T is reckoned from noon. The solution gives also a small sine term, with coefficient $0''.03$, which does not appear to be distinct enough to adopt. The cosine term is less than one half the difference in the observed clock corrections at sunset and sunrise. It is of the same size approximately as that of the fourteen-month term of the latitude variation.

Since the axis of figure of the earth does not coincide with the axis of rotation of the earth, the pole of figure makes a daily revolution about the pole of rotation. The pole of figure advances only $0^\circ.8$ on its curve, representing the fourteen-month rotation, so the daily revolution will be nearly circular.

If the deviation of the two axes is constant during a day, there could be no resulting variation of latitude, according to the accepted definition of that coördinate, assuming that the axis of rotation does not shift its position during the same period.

If the position of the instantaneous axis of rotation of the earth with respect to the celestial sphere still requires correction, depending upon an error in the adopted constant of nutation, these several anomalies of observation may possibly be reconciled with theory.

As observations of this character have served to determine our astronomical constants, modern refined observations may indicate a need of revision of the values.

The test should be sought in fundamental observations with the meridian circle, since the full amplitude of variations can be more effectually observed, while other classes of observation have given mainly tests of differential changes.

Since aberration has minimum effects upon transits of stars near sunset and sunrise, and also minimum effects upon zenith distances of stars near noon and midnight, the two diurnal variations in our results do not appear to indicate any correction to the constant of aberration.

The diurnal term in observed ϕ may indicate a small correction ($0''.02$) to the constant of nutation.

The solution of a diurnal term is commonly involved with that of an annual periodic term, when dealing with observations of any star at all hours of the day, but the observations of *Polaris* include many consecutive transits at opposite culminations.

The variation in clock corrections or rates is distinctly of a diurnal character.

DIURNAL TERM IN LATITUDE VARIATION.

Epoch		Δ φ	Comp.	O–C
$0^h.6$	P.M.	+0″.06	+0″.14	−0″.08
1 .4	"	+0 .07	0 .13	−0 .06
2 .5	"	+0 .03	0 .11	−0 .08
3 .4	"	+0 .08	0 .09	−0 .01
4 .4	"	+0 .04	0 .06	−0 .02
5 .5	"	−0 .30	+0 .02	−0 .32
6 .6	"	+0 .26	−0 .02	+0 .28
7 .3	"	−0 .08	0 .05	−0 .03
8 .5	"	−0 .07	0 .09	+0 .02
9 .4	"	+0 .02	0 .11	+0 .13
10 .4	"	−0 .15	0 .13	−0 .02
11 .6	"	−0 .10	0 .14	+0 .04
0 .4	A.M.	−0 .30	0 .14	−0 .16
1 .5	"	−0 .34	0 .13	−0 .21
2 .4	"	−0 .14	0 .11	−0 .03
3 .3	"	−0 .01	0 .09	+0 .08
4 .3	"	+0 .09	0 .06	+0 .15
5 .2	"	+0 .01	−0 .03	+0 .04
6 .5	"	+0 .12	+0 .02	+0 .10
7 .4	"	+0 .09	0 .05	+0 .04
8 .4	"	+0 .10	0 .08	+0 .02
9 .3	"	+0 .11	0 .11	00
10 .5	"	+0 .26	0 .13	+0 .13
11 .3	"	+0 .14	0 .14	00
Mean		0 .00	0 .00	0 .00
Average		±0 .12	±0 .09	±0 .08

$$\Delta\,\phi = +0''.14 \cos T$$

The groups of Δ φ have an average above 42 observations each.

The probable error of the night groups derived from individual residuals is ±0″.08, and that of the day groups is ±0″.12, average ±0″.10.

The two sunset groups have residuals of four times the average O–C, which would be ±0″.06 if these two results are combined into one.

Observations.

Zenith stars	258
α *Andromedæ*	135
Polaris	231
β *Ursæ Minoris*	77
Groups	319

The latitude from this partial summation of our fundamental observations is $\phi_0 = 37° 20' 25''.6$. This was the value derived from the first effective meridian circle work here. Previous to the beginning of that work, in 1893, the adopted latitude of the instrument, as furnished by the *U. S. Coast and Geodetic Survey,* was a full second of arc smaller. This difference could be due to the errors of declination of the stars employed in the earlier determination, and to the lack of corrections for the periodic variation of latitude.

As to possible sources of systematic error in our zenith distance observations, there might be a sensible difference between nadir readings during daytime and night.

This would probably not appear as a periodic term, however. All accidental errors, even those of circle readings, are larger in the daytime.

It is more difficult to concede the probability of a shift in the zenith point, due to a variation in the refraction at the zenith. Especially is this less probable during the night hours, when the atmospheric conditions are most stable.

Whether our observed periodic variation is in the clock rates, or in some term affecting the position of the meridian, the determination of the difference of longitude between two widely separated stations will show the effect if the phase is the same for both stations.

The usual procedure for an exchange of longitude signals is to observe the same list of stars at each station, in order to eliminate errors of right ascension.

If the stations differ 90° in longitude, the rate of one clock will be carried forward six hours to the epoch of observation at the other station.

If the established daily rate of the clock be used, the accumulated error would amount to $0^s.03$ in this interval.

Every exchange of signals would have this error, but if signals are sent each six hours during one day the errors would occur in pairs, two successive plus errors being followed by two minus errors.

The double amplitude of $0^s.06$ would occur in two consecutive exchanges, twelve hours apart.

If two stations differ 180° in longitude, exchange of signals every six hours would give differences alternately of $0^s.06$ and zero.

If we take an exchange of signals between Greenwich and a station 90° west, and the group of stars is observed at midnight, the derived difference of longitude would be $0^s.03$ in error, but no difference in the exchanges at both epochs of observation would appear. If the group of stars were observed at sunset at each station, the errors of projected clock rates would be of contrary signs, and there should be a difference of $0^s.06$ in the exchanges.

Δ Longitude 90°.

W.	Corr.	E.	Corr.	W.—E.	Change.
6 P.M.	$+0^s.03$	Midnight	^{s} 00	$+0^s.03$	
					^{s} 00
Midnight	00	6 A.M.	-0.03	$+0.03$	
					-0.06
6 A.M.	-0.03	Noon	00	-0.03	
					00
Noon	00	6 P.M.	$+0.03$	-0.03	
					$+0.06$
6 P.M.	$+0.03$	Midnight	00	$+0.03$	

Δ Longitude 180°.

W.	Corr.	E.	Corr.	W.—E.	Change.
6 P.M.	$+0^s.03$	6 A.M.	$-0^s.03$	$+0^s.06$	
					$-0^s.06$
Midnight	00	Noon	00	00	
					-0.06
6 A.M.	-0.03	6 P.M.	$+0.03$	-0.06	
					$+0.06$
Noon	00	Midnight	00	00	
					$+0.06$
6 P.M.	$+0.03$	6 A.M.	-0.03	$+0.06$	

Δ Longitude 120°.

W.	Corr.	E.	Corr.	W.—E.	Change.
6 P.M.	$+0^s.03$	2 A.M.	$-0^s.01$	$+0^s.04$	
					$-0^s.02$
Midnight	00	8 A.M.	-0.02	$+0.02$	
					-0.06
6 A.M.	-0.03	2 P.M.	$+0.01$	-0.04	
					$+0.02$
Noon	00	8 P.M.	$+0.02$	-0.02	
					$+0.06$
6 P.M.	$+0.03$	2 A.M.	-0.01	$+0.04$	

If the phase of variation has a constant epoch, the maxima would occur at the same instant for all stations, and the exchange of signals would exhibit no changes.

Without regard to the true difference in longitude, an exchange of clock signals at intervals of six hours through one day, between two stations 90° apart, may furnish a test of the phase of the variation, if it is in clock rates. The only necessary conditions are two clocks with well-established uniform daily rates and the requisite precision in recording the signals.

It is hoped, and even hopefully expected, that a transatlantic record of the wireless signals from Bordeaux may be employed for this purpose.

Our current series of observations should have a weight of ten per night, as compared with the earlier work tabulated above, since there are about eighty stars observed in each period of six hours, while most of the earlier work included but eight stars in periods of four hours.

Whatever the result, when we close the cycle of observations next September the weight will be assumed equal to that of all the preceding observations. This policy is justified by the rigorous attention that is being paid to the necessary details of program and of reduction. An interchange of stars or the loss of a single observation will always mask, if not mar, the effect of the small correction we are sifting out. Nevertheless there has been, personally, a convincing effect in the weight of evidence of the old work, from which these variations emerge as a by-product—to close with an industrial figure of speech.

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